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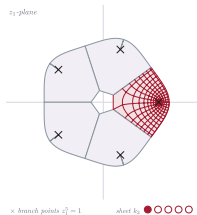
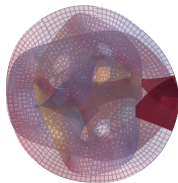


Homological Mirror Symmetry I

Two geometries of one torus

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Two geometries that compute the same numbers

A-side (symplectic)

A manifold with a closed non-degenerate 2-form ω . Invariants: *counts of holomorphic curves*, Lagrangian submanifolds, Floer cohomology. Depends on ω , not on the complex structure.

B-side (complex)

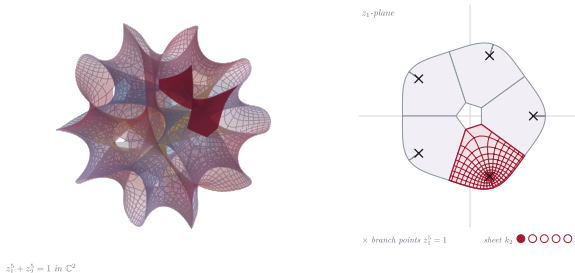
A complex manifold or algebraic variety. Invariants: *periods*, Hodge structures, coherent sheaves. Depends on the complex structure, not on ω .

Mirror symmetry pairs Calabi–Yau manifolds $X, \check{X}$ so that the A-side of X is the B-side of $\check{X}$ and vice versa. The first evidence was numerical:

Candelas–de la Ossa–Green–Parkes 1991. The number of rational curves of degree d on the quintic threefold (2875 lines, 609250 conics, ...) is read off from the periods of the mirror quintic: an A-side count computed by B-side integrals.

The quintic, seen through a slice

The quintic $\{\sum z_i^5 = 0\} \subset \mathbb{CP}^4$ is a Calabi–Yau threefold. A slice we can see: the Fermat curve $C = \{z_1^5 + z_2^5 = 1\} \subset \mathbb{C}^2$, a Riemann surface of genus 6.



Left (after Hanson): 25 patches $z_1 = e^{2\pi i k_1/5} \cosh(w)^{2/5}$, $z_2 = e^{2\pi i k_2/5} (-i \sinh w)^{2/5}$, projected $\mathbb{R}^4 \rightarrow \mathbb{R}^3$. Right: $(z_1, z_2) \mapsto z_1$ is 5-sheeted, branched over $z_1^5 = 1$; the lit patch is one sheet.

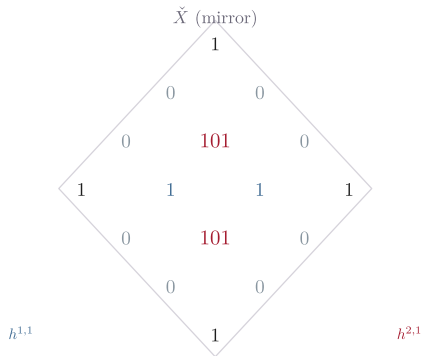
The first symptom: Hodge numbers trade places

For a Calabi–Yau threefold, $h^{1,1}$ counts Kähler deformations (**A-side**), $h^{2,1}$ counts complex deformations (**B-side**). A mirror pair satisfies

$$h^{p,q}(X) = h^{3-p,q}(\check{X}),$$

so $h^{1,1}(X) = h^{2,1}(\check{X})$. Together with Hodge symmetry $h^{p,q} = h^{q,p}$, the mirror diamond is the original one turned by a quarter.

The quintic. $h^{1,1} = 1, h^{2,1} = 101$: one Kähler class, 101 complex moduli. Its mirror has $h^{1,1} = 101, h^{2,1} = 1$.



Kontsevich's question, and the plan

Numbers match because *categories* match. Kontsevich (ICM 1994) proposed that mirror symmetry is an equivalence of triangulated categories, one built from Lagrangians, the other from sheaves.

Lecture I. Build both categories from scratch. The A-side: Floer cohomology and the Fukaya category. The B-side: coherent sheaves. Check the conjecture on the elliptic curve, triangle by triangle. Then explain *why* mirrors exist: torus fibrations and tropical geometry.

Lecture II. Leave the Calabi–Yau world: Fano manifolds and their Landau–Ginzburg mirrors, matrix factorizations, Knörrer periodicity, hypersurfaces and pairs of pants, and the Fukaya category of a *singular* hypersurface [Jeffs 2023].

1

The A-side: Lagrangians and Floer theory

Symplectic manifolds and Lagrangians

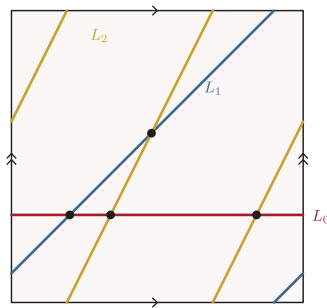
Definition

A symplectic manifold (M^{2n}, ω) : $d\omega = 0$ and $\omega^n \neq 0$. A submanifold $L^n \subset M$ is *Lagrangian* if $\omega|_L = 0$.

Darboux: locally $\omega = \sum dx_i \wedge dy_i$; all invariants are global. On a surface every curve is Lagrangian. On the torus $T^2 = \mathbb{R}^2/\mathbb{Z}^2$ with $\omega = dx \wedge dy$, the line of slope (a, b) through a point closes up to a circle $L_{(a,b)}$.

Count

$$\#(L_{(a,b)} \cap L_{(c,d)}) = |ad - bc|.$$



Floer cohomology: intersections, corrected

For transverse L_0, L_1 let

$$CF(L_0, L_1) = \bigoplus_{p \in L_0 \cap L_1} \Lambda p \ (\Lambda \text{ the Novikov field}),$$

and

$$\mu^1(p) = \sum_q \sum_{[u] \in \mathcal{M}(p, q)} \pm T^{\omega(u)} q,$$

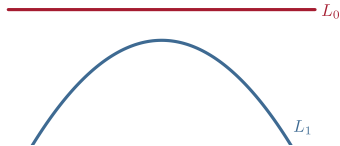
counting rigid holomorphic strips

$u : \mathbb{R} \times [0, 1] \rightarrow M$ from p to q with boundary on L_0 and L_1 .

Floer 1988

If ω vanishes on $\pi_2(M, L_i)$, then $\mu^1 \circ \mu^1 = 0$ and $HF(L_0, L_1)$ is invariant under Hamiltonian isotopy.

$$HF(L_0, L_1) = 0$$



p and q bound one strip, so $\mu^1 p = \pm q$: they cancel in cohomology, and they disappear together under the isotopy.

Floer theory near L is Morse theory

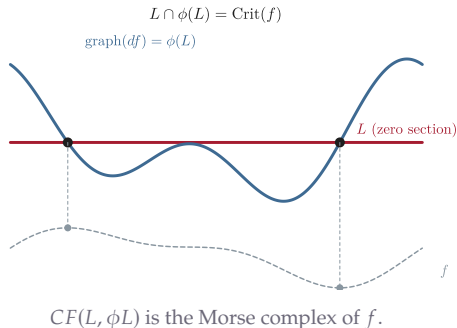
Weinstein neighbourhood

A neighbourhood of a Lagrangian L is symplectomorphic to a neighbourhood of the zero section in T^*L .

A small Hamiltonian push-off of L is $\text{graph}(df)$, f Morse: $L \cap \phi(L) = \text{Crit}(f)$, and the strips are gradient lines of f [Floer 1988].

Consequence

For exact L : $HF(L, L) \cong H^*(L)$; otherwise disks correct it.



Why $\mu^1 \circ \mu^1 = 0$: count the ends

Proof idea. The coefficient of r in $\mu^1 \mu^1(p)$ counts broken strips $p \rightarrow q \rightarrow r$. These are exactly the boundary points of the compactified 1-dimensional moduli space $\overline{\mathcal{M}}(p, r)$ (Gromov compactness: energy is bounded by $\omega(u)$, and with no disks or spheres to bubble off, the only degeneration is breaking). A compact 1-manifold has an even number of boundary points, counted with signs they cancel: $\mu^1 \mu^1 = 0$.

Invariance. A Hamiltonian isotopy ϕ_t gives a continuation map $CF(L_0, L_1) \rightarrow CF(L_0, \phi_1 L_1)$ counting strips with moving boundary; the same boundary argument shows it is a chain map with a homotopy inverse.

Corollary (Arnold–Givental type)

If ϕ is Hamiltonian and $L \pitchfork \phi(L)$ for an exact L , then $\#(L \cap \phi(L)) \geq \dim HF(L, L) = \dim H^*(L)$.

Moduli spaces of disks, and what can go wrong

Moduli space

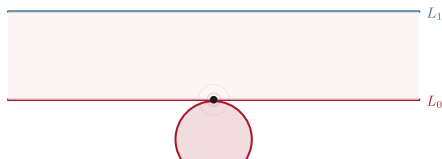
$\mathcal{M}(p, q; \beta)$: strips from p to q in class β , modulo $\mathbb{R}$; for generic J a manifold of dimension $\mu(\beta) - 1$ (μ : Maslov index).

Gromov compactness

Bounded energy: limits exist up to breaking, *disk bubbles* and sphere bubbles.

Bubbling is why Floer needs $\omega|_{\pi_2(M,L)} = 0$.

strip + disk bubble



Gradings: the Maslov index

A loop of Lagrangian planes $\Lambda(\theta) \subset \mathbb{C}^n$ has a winding number: write $\Lambda = U\mathbb{R}^n$ with U unitary; $\det^2 U \in S^1$ is well defined, and

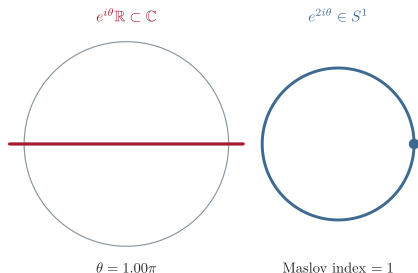
$$\mu(\Lambda) = \deg(\theta \mapsto \det^2 U(\theta)).$$

For the line $e^{i\theta}\mathbb{R} \subset \mathbb{C}$, $\theta \in [0, \pi]$: $\mu = 1$.

Graded Lagrangians

If $2c_1(M) = 0$, a choice of grading on L (a lift of the phase $\det^2$ to $\mathbb{R}$) gives each $p \in L_0 \cap L_1$ an integer degree, and μ^1 has degree 1: CF becomes $\mathbb{Z}$ -graded.

On the B-side this is the cohomological degree of Ext^i .



Polygons and the A_∞ relations

Disks with $k + 1$ boundary punctures and boundary on $L_0, \dots, L_k$ define

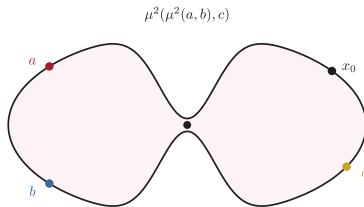
$$\mu^k : CF(L_{k-1}, L_k) \otimes \cdots \otimes CF(L_0, L_1) \rightarrow CF(L_0, L_k).$$

μ^2 is associative only up to homotopy; the ends of 1-dimensional moduli spaces of disks give

$$\sum_{i+j+l=k} \pm \mu^{i+1+l}(\text{id}^{\otimes i} \otimes \mu^j \otimes \text{id}^{\otimes l}) = 0.$$

The Fukaya category $\mathcal{F}(M)$

Objects: graded spin Lagrangians. Morphisms: CF . Composition: the μ^k .



The disks with four punctures form an interval; its two ends are the two bracketings.

2 *The B-side: coherent sheaves*

The derived category of coherent sheaves

$D^b \text{Coh}(X)$

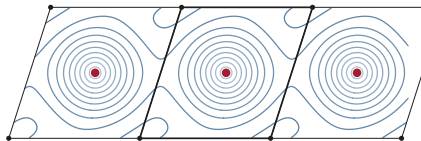
Objects: complexes of coherent sheaves (vector bundles, structure sheaves of subvarieties, skyscrapers). Morphisms: $\text{Ext}^i(E, F)$; for bundles $\text{Ext}^i(E, F) = H^i(X, E^\vee \otimes F)$.

On $E_\tau = \mathbb{C}/(\mathbb{Z} + \tau\mathbb{Z})$, Riemann–Roch and Serre duality give, for $\deg \mathcal{L} > 0$,

$$\dim H^0(\mathcal{L}) = \deg \mathcal{L}, \quad H^1(\mathcal{L}) = 0.$$

The degree-one bundle has one section up to scale: the theta function, with its zero at $\frac{1+\tau}{2}$.

$$E_\tau = \mathbb{C}/(\mathbb{Z} + \tau\mathbb{Z})$$



$$\vartheta(z) = \sum_n e^{\pi i n^2 \tau + 2\pi i n z}$$

$|\vartheta(z)| e^{-\pi(\text{Im } z)^2 / \text{Im } \tau}$ is doubly periodic: its level sets around the single zero in each fundamental domain.

Computing Ext : three examples on a curve

A skyscraper with itself. On a curve, $\mathcal{O}_p$ has the resolution $0 \rightarrow \mathcal{O}(-p) \xrightarrow{s} \mathcal{O} \rightarrow \mathcal{O}_p \rightarrow 0$. Applying $\text{Hom}(-, \mathcal{O}_p)$: both maps vanish at p , so $\text{Ext}^0 = \mathbb{C}$, $\text{Ext}^1 = \mathbb{C}$:

$$\text{Ext}^*(\mathcal{O}_p, \mathcal{O}_p) = \mathbb{C} \oplus \mathbb{C}[-1] = \Lambda^* T_p.$$

Line bundles on E_τ . $\text{Ext}^i(\mathcal{L}_a, \mathcal{L}_b) = H^i(\mathcal{L}_{b-a})$: dimension $b-a$ in degree 0 if $b > a$, in degree 1 if $b < a$ (Serre duality, $K_E \cong \mathcal{O}$); for $b = a$, $H^*(\mathcal{O}) = \mathbb{C} \oplus \mathbb{C}[-1]$.

Serre duality is Calabi–Yau

$\text{Ext}^i(E, F) \cong \text{Ext}^{n-i}(F, E \otimes K_X)^\vee$, and $K_X \cong \mathcal{O}_X$ makes $D^b\text{Coh}(X)$ an n -Calabi–Yau category. On the A-side: Poincaré duality $HF^i(L_0, L_1) \cong HF^{n-i}(L_1, L_0)^\vee$.

Mirror check. $\text{Ext}^*(\mathcal{O}_p, \mathcal{O}_p) = \mathbb{C} \oplus \mathbb{C}[-1] = H^*(S^1) = HF^*(L, L)$ for the mirror circle $L_{(0,1)}$; and $\mathcal{L}_a \rightarrow \mathcal{L}_b$ in degree 1 for $b < a$ is the A-side grading of an intersection of slopes $a > b$.

3 *The first example: $\mathbb{C}^*$*

The A-side of $\mathbb{C}^*$: wrapping creates z^k

$\mathbb{C}^* \cong T^*S^1$; the key object is the non-compact fibre $F = \{\theta = \theta_0\}$.

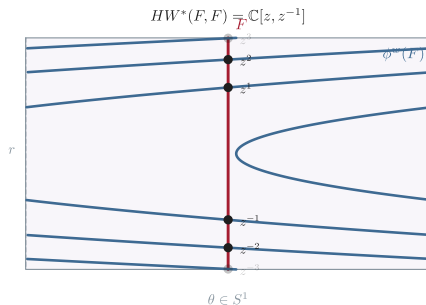
Wrapped Floer cohomology

$$HW^*(F, F) = \varinjlim_w HF^*(F, \phi_H^w(F)), \quad H = r^2/2 \text{ rotating the ends.}$$

Each wrap adds x_k, x_{-k} , all in degree 0.

Products

$$\mu^2(x_a, x_b) = x_{a+b} \text{ (one triangle each), so } HW^*(F, F) \cong \mathbb{C}[z, z^{-1}] \text{ with } x_k = z^k.$$



Theorem (Abouzaid; Abouzaid–Seidel)

The fibre F generates the wrapped Fukaya category, and

$$D^\pi \mathcal{W}(T^*S^1) \simeq \text{Perf } \mathbb{C}[z^{\pm 1}] = D^b \text{Coh}(\mathbb{C}^*).$$

Proof idea. A generator on each side with the same endomorphism algebra suffices: $\text{End}(F) = HW^*(F, F) = \mathbb{C}[z^{\pm 1}]$ and $\text{End}(\mathcal{O}_{\mathbb{C}^*}) = \mathbb{C}[z^{\pm 1}]$, with no higher products ($\mu^k = 0, k \geq 3$, by degree).

A-side: T^*S^1

B-side: $\mathbb{C}^*$

fibre F

$\mathcal{O}_{\mathbb{C}^*}$

S^1 + holonomy λ

skyscraper $\mathcal{O}_\lambda$

$HW(F, F)$

$\mathbb{C}[z^{\pm 1}]$

$HF(F, S^1_\lambda) = \mathbb{C}$

$\text{Hom}(\mathcal{O}, \mathcal{O}_\lambda) = \mathbb{C}$

Used in Lecture II. The Dehn twist of the cylinder acts on this algebra by multiplication by $1+z$; inverting it is Auroux's definition of the Fukaya category of the node.

Conjecture (Kontsevich 1994)

For mirror Calabi–Yau manifolds there is an equivalence of triangulated categories

$$D^{\pi}\mathcal{F}(X) \simeq D^b\mathrm{Coh}(\check{X}),$$

and symmetrically $D^{\pi}\mathcal{F}(\check{X}) \simeq D^b\mathrm{Coh}(X)$ (D^{π} : split-closed twisted complexes).

Why this explains the numbers. Hochschild homology recovers $H^*(X)$ and $\bigoplus H^q(\Omega_{\check{X}}^p)$: Hodge numbers match; deformations match the moduli, giving the enumerative predictions.

Proved for: elliptic curves [Polishchuk–Zaslow 1998], quartic surfaces [Seidel 2003], abelian varieties, Calabi–Yau hypersurfaces in projective space [Sheridan 2015], and many non-compact and Fano examples (Lecture II).

4

The elliptic curve, worked out

The torus (T^2, ω_τ) with complexified area τ is mirror to E_τ .

A-side: (T^2, ω_τ)

B-side: E_τ

$L_{(1,d)}$, slope d

$\mathcal{L}_d$, degree d

$L_{(0,1)}$ + flat connection

skyscraper $\mathcal{O}_z$

$L_{(1,0)} \cap L_{(1,d)}$

basis of $H^0(\mathcal{L}_d)$

triangles, weighted by area

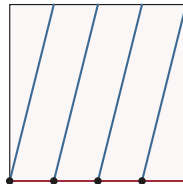
products of thetas

Dehn twist along $L_{(0,1)}$

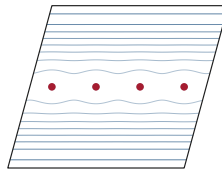
$\otimes \mathcal{L}_1$

First check: dimensions. $\#(L_{(1,0)} \cap L_{(1,d)}) = d = h^0(\mathcal{L}_d)$.

$L_{(1,4)} \cap L_0 : 4 \text{ pts}$



$\vartheta(4z, 4\tau) : 4 \text{ zeros}$



$\deg \mathcal{L}_4 = 4$

$L_{(1,d)}$ meets L_0 in d points; the section $\vartheta(dz, d\tau)$ of $\mathcal{L}_d$ has d zeros ($d = 1, \dots, 4$).

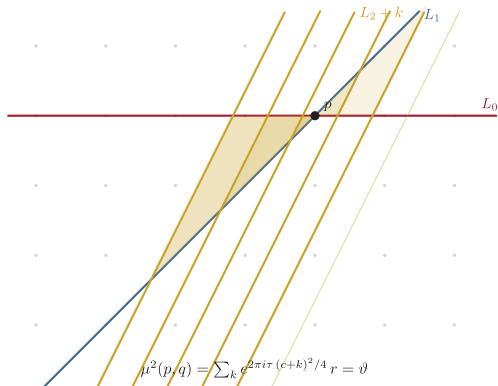
Triangles are theta functions

Lift to $\mathbb{R}^2$: $L_0 = \{y = 0\}$, $L_1 = \{y = x\}$, and L_2 has lifts $\{y = 2x + c + k\}$, $k \in \mathbb{Z}$. Each k gives one triangle at $p = 0$, of area

$$A_k = \frac{1}{4}(c + k)^2.$$

Polishchuk–Zaslow 1998

$\mu^2(p, q) = \sum_{k \in \mathbb{Z}} e^{2\pi i \tau (c+k)^2/4} r$, a theta function with characteristic c : exactly the structure constant of $H^0(\mathcal{L}_1) \otimes H^0(\mathcal{L}_1) \rightarrow H^0(\mathcal{L}_2)$.



Full proof I (B-side): the square of the theta function

Claim. Let $\theta(z) = \sum_m e^{\pi i m^2 \tau + 2\pi i m z}$ (the section of $\mathcal{L}$) and $\theta_k(w) = \sum_l e^{\pi i (2\tau)(l + \frac{k}{2})^2 + 2\pi i w(l + \frac{k}{2})}$, $k = 0, 1$ (a basis of $H^0(\mathcal{L}^2)$ in $w = 2z$). Then $\theta(z)^2 = \sum_{k=0,1} \theta_k(0) \theta_k(2z)$.

1 **Expand.** Both factors converge absolutely for $\text{Im } \tau > 0$, so

$$\theta(z)^2 = \sum_{m_1, m_2 \in \mathbb{Z}} e^{\pi i \tau (m_1^2 + m_2^2) + 2\pi i z (m_1 + m_2)}.$$

2 **New variables.** Put $s = m_1 + m_2$, $d = m_1 - m_2$; then $s \equiv d \pmod{2}$. Let $k = d \pmod{2} \in \{0, 1\}$ and $s = 2l + k$, $d = 2j + k$.

3 **Bijection.** $(m_1, m_2) \mapsto (l, j, k)$ is a bijection $\mathbb{Z}^2 \rightarrow \mathbb{Z}^2 \times \{0, 1\}$ with inverse $m_1 = l + j + k$, $m_2 = l - j$ (check: $s = 2l + k$, $d = 2j + k$).

4 **Quadratic form.** $m_1^2 + m_2^2 = \frac{1}{2}(s^2 + d^2)$, and

$$\frac{1}{2}(2l + k)^2 = 2(l + \frac{k}{2})^2, \quad \frac{1}{2}(2j + k)^2 = 2(j + \frac{k}{2})^2.$$

Linear term: $m_1 + m_2 = s = 2(l + \frac{k}{2})$.

5 **Substitute.** The exponent becomes

$$\begin{aligned} & \pi i (2\tau) (l + \frac{k}{2})^2 + 2\pi i (2z) (l + \frac{k}{2}) \\ & + \pi i (2\tau) (j + \frac{k}{2})^2. \end{aligned}$$

6 **Factorise.** The sum over (l, j) splits for each k : for fixed k the l -sum is $\theta_k(2z)$ (step 5, first line) and the j -sum is $\theta_k(0)$ (second line):

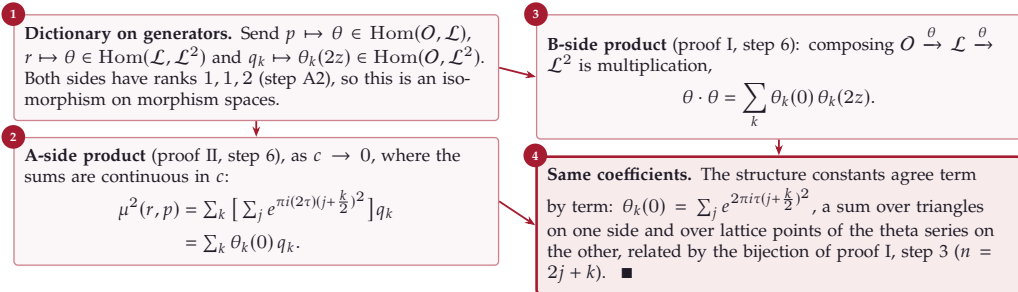
$$\theta(z)^2 = \sum_{k=0,1} \theta_k(2z) \theta_k(0). \quad \blacksquare$$

Full proof II (A-side): counting the triangles

- 1 **Setup.** $T^2 = \mathbb{R}^2/\mathbb{Z}^2$ with complexified Kähler form $B + i\omega$, $\int_{T^2} = \tau$. A holomorphic disk u contributes $e^{2\pi i \tau \text{Area}(u)}$, area measured in units of T^2 . Take $L_0 = \{y = 0\}$, $L_1 = \{y = x\}$, $L_2 = \{y = 2x + c\}$, $0 < c < 1$ (slope = degree; the shift c makes them transverse).
- 2 **Lifts and intersections.** Translating by $(m_1, m_2) \in \mathbb{Z}^2$, the lifts of L_2 are $\ell_n = \{y = 2x + c + n\}$ with $n = m_2 - 2m_1$ ranging over all of $\mathbb{Z}$. In T^2 : $L_0 \cap L_1 = \{p\}$, $L_1 \cap L_2 = \{r\}$ ($x = -(n+c)$, one class), $L_0 \cap L_2 = \{q_0, q_1\}$ ($x = -\frac{n+c}{2} \bmod 1$, q_k for $n \equiv k$). Ranks $1, 1, 2 = h^0(\mathcal{L}), h^0(\mathcal{L}), h^0(\mathcal{L}^2)$.
- 3 **Lift the triangle.** A holomorphic triangle $u : D \rightarrow T^2$ with corners p, r, q_k lifts to $\mathbb{R}^2$ (D is simply connected). Fix the lift with corner p at the origin: its edges lie on $y = 0$, $y = x$ and one ℓ_n .
- 4 **One triangle per n .** The three lines bound the straight triangle Δ_n with vertices $0, (t, 0), (2t, 2t)$, $t = -\frac{n+c}{2}$. By the open mapping theorem and the maximum principle the image of u is Δ_n , and by the Riemann mapping theorem Δ_n is the image of exactly one holomorphic triangle up to reparametrisation.
- 5 **Area and sign.** Signed area $= \frac{1}{2} \det \begin{pmatrix} t & 2t \\ 0 & 2t \end{pmatrix} = t^2 > 0$ for every n : all triangles have the same orientation, so all count with sign $+1$, and $\text{Area}(\Delta_n) = \left(\frac{n+c}{2}\right)^2$.
- 6 **Sum.** Write $n = 2j + k$; then Δ_n ends at q_k with area $\left(j + \frac{k+c}{2}\right)^2$, and

$$\mu^2(r, p) = \sum_{k=0,1} \left[\sum_{j \in \mathbb{Z}} e^{2\pi i \tau (j + \frac{k+c}{2})^2} \right] q_k.$$

Full proof III: the two sides agree



What the proof shows. One Gaussian sum, twice: triangle areas on the A-side, the diagonalised form $m_1^2 + m_2^2$ on the B-side. General c gives characteristic $\frac{k+c}{2}$; higher μ^k match Massey products [Polishchuk–Zaslow 1998].

Proof idea. Every holomorphic triangle with corners at fixed intersection points lifts to $\mathbb{R}^2$ and is a straight triangle, so the count is a sum over lattice translates k of one line; the area is quadratic in k , which makes the weighted count a Gaussian sum. On the B-side, multiplying two theta functions of degree one and expanding in the basis of degree-two theta functions gives the same Gaussian sums (the classical addition formula).

Higher products. μ^3 counts quadrilaterals and matches Massey products on E_τ ; with them the comparison extends to an equivalence of derived categories [Polishchuk–Zaslow; Fukaya; Abouzaid–Smith].

What the example teaches. Slopes become degrees, intersections become sections, areas become exponents. The torus fibration $T^2 \rightarrow S^1$ by the circles $L_{(0,1)}$ is what organises it.

5

Why mirrors exist: torus fibrations

SYZ: mirror symmetry is T-duality

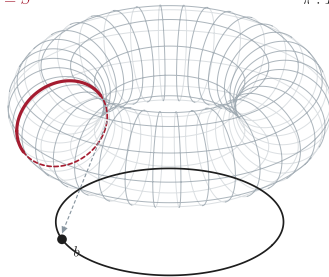
Conjecture (Strominger–Yau–Zaslow 1996)

Mirror Calabi–Yau manifolds carry special Lagrangian torus fibrations over the same base, $X \rightarrow B \leftarrow \check{X}$, whose regular fibres are dual tori: $\check{T} = H^1(T; U(1))$.

Where the dictionary comes from. A point of $\check{X}$ is a fibre T of X with a flat $U(1)$ connection. A section becomes a line bundle, a fibre a skyscraper.

$$\pi^{-1}(b) = S^1$$

$$\pi : T^2 \rightarrow S^1$$



For the elliptic curve the base is a circle and the fibres are the circles $L_{(0,1)}$.

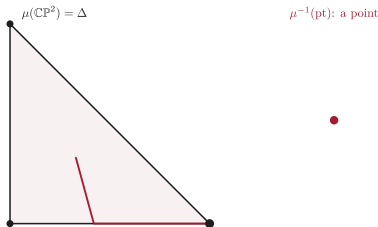
Toric geometry: the fibration you can see

$(\mathbb{C}^*)^n$ is fibred by tori over $\mathbb{R}^n$:

$$\text{Log} : (\mathbb{C}^*)^n \rightarrow \mathbb{R}^n, \quad z \mapsto (\log |z_1|, \dots, \log |z_n|),$$

with fibre T^n . A toric variety compactifies this: the moment map sends $\mathbb{CP}^2$ onto a triangle; over the interior the fibre is T^2 , over an edge S^1 , over a vertex a point.

Towards Lecture II. The boundary of the polytope is where the fibration degenerates; for Fano X it is recorded on the mirror by a function, the superpotential W .



The fibre over a point moving to an edge, then to a vertex: $T^2 \rightarrow S^1 \rightarrow \text{pt}$.

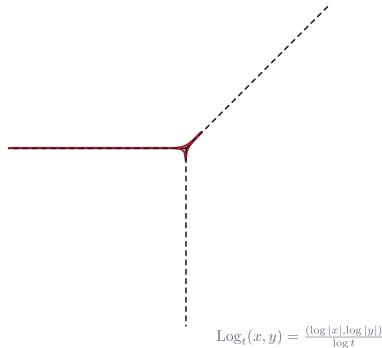
The *amoeba* of $H = \{f = 0\} \subset (\mathbb{C}^*)^n$ is $\text{Log}(H)$.
For the line $1 + x + y = 0$: the points
 $(\log r, \log |1 + re^{i\theta}|)$, with three tentacles.

Mikhalkin, Rullgård

For $f_t = \sum c_\alpha t^{\varphi(\alpha)} z^\alpha$, the rescaled amoebas $\text{Log}_t(H_t)$ converge (Hausdorff) to the tropical hypersurface: the corner locus of $\max_\alpha (\langle \alpha, x \rangle - \varphi(\alpha))$.

For $1 + x + y$: the corner locus of $\max(0, x, y)$.

$\log t = 12.00$



Summary of Lecture I

A-side

Lagrangians, Floer cohomology from holomorphic strips, A_∞ products from polygons; all identities come from counting the ends of 1-dimensional moduli spaces.

B-side

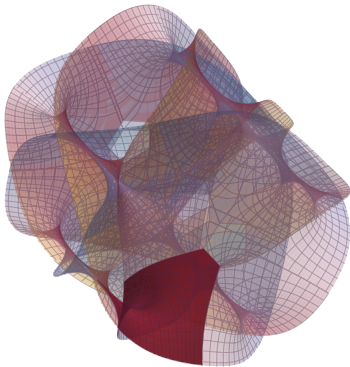
Coherent sheaves and their Ext groups; on E_τ , theta functions.

HMS

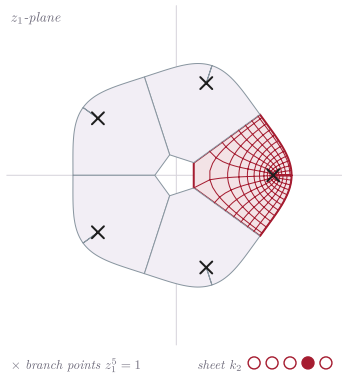
$D^\pi \mathcal{F}(X) \simeq D^b \text{Coh}(\check{X})$; on the elliptic curve every structure constant is a theta function, because triangle areas are quadratic in lattice translates.

Next time. Fano manifolds, Landau–Ginzburg models and Lefschetz fibrations; matrix factorizations; pairs of pants; and what the Fukaya category of a singular hypersurface should be.

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$$z_1^5 + z_2^5 = 1 \text{ in } \mathbb{C}^2$$



Thank you.

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